Systematic inventory management with a computer

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TILBURG INSTITUTE OF ECONOMICS
DEPARTMENT OF ECONOMICS

SYSTEMATIC INVENTORY MANAGEMENT WITH A COMPUTER

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+ inventory models

MAY 1972

DEPARTMENT OF ECONOMETRICS

I. SYSTEMATIC INVENTORY MANAGEMENT WITH A COMPUTER. *) **)

1. Introduction.

The usual arguments of managers to contact you about their inventorv problems run somewhat as follows: we have high stocks and feel that too much of our capital is tied up in it, but nevertheless too manv times, goods are not available if we need them, so too many times there is a shortage. Though these complaints mav sound contradictorv, thev are heard at the same time verv often. The explanation is not difficult to give: one has too high stocklevels for some articles and too low levels for other ones. The most important question is: how to find a systematic approach to tackle these tvpes of problems?
One of the most succesful ways of attack is to make a mathematical model of the situation. However, if one wants to make a mathematical model of a situation, it is necessarv to make explicit assumptions about the process we are studving and this again can onlv be done in a good wav if one is able to give a precise verbal description of the process. Now, what is happening if one is keeping goods in stock? Verv roughlv one coeld give the following picture: buving or producing -storing- selling or using (articles). It is interesting to realise that this picture holds true for quite different situations, e.g. the case of raw materials in a factory; the case of having spare parts in stock, to be able to repair machines which suddenly fail, rapidlv; the case of a whole -sale dealer who is buing, storing and selling the products, he carries, but also the case of a physician or a dentist who is asking his patients to visit him according to a schedule, designed in advance.

*) Summarv of lectures presented during a graduate course in Management Science at the Universitv of Novi Sad in Mav 1972.

**) The author is indebted to G.R. Mustert for his critical remarks.

The picture of the situation, given above, is still too vague to make good assumptions for the mathematical model. A somewhat more detailed description might run as follows; to be precise, we now focus our attention to the case of a wholesale dealer. We discern the following aspects to be taken into account.

Aspects	Possible assumptions
1. ordertimes	ordertimes are fixed (e.g. once a week)
	" " free (one can order when one wants to do so)
2. purchasing costs	a constant C_o *) (setup costs) + costs proportional to number of units bought (=q)
	C_o + costs not proportional to number of units bought
3. leadtime	zero or to be neglected
	constant
	variable with a distribution function
4. costs of storing	usually assumed to be proportional to number of units in stock and to time stocked: C_1 per unit, per time unit
5. demand	constant per unit of time
	variable " " " " with a distribution function

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*) e.g. costs in a purchasing department
" " an administrative department

6. costs of shortages
- proportional to number of units and to time there is a shortage (the case of subsequent deliveries)
- proportional to number of units only (the case of lost demand)

The argument to describe the inventory problem carefully was that we wanted to construct a mathematical model in order to minimize total costs, which in this case consist of three components: purchasing costs, stock-holding costs and costs because of shortages. These total costs can be influenced by [1] the times at which order and [2] the amounts we order. So in this respect, there are two questions to be answered:

1) when should we give a new order to our supplier,

2) if we order, how much should be ordered.

These questions can be answered if we first construct a mathematical model, starting from some assumptions. It will be clear that starting from different assumptions may lead to different answers, to both questions. In order to give some idea in what way these answers might be developed, we will next discuss two simple examples.

2. Two simple models*)

Model 1

Assumptions as to the aspects mentioned in section 1.

1. free
2. C_o + costs proportional to q
3. zero
4. C_1 per unit, per unit of time
5. constant, r units per unit of time
6. not allowed (this assumption can be made, because of assumption 3).

If we minimize total costs in this model, we balance setup or orderingcosts against stock holding costs.

The question, when to order, is easily answered: we order if the stocklevel is zero and if there is demand. To answer the second question, we make the following assumptions.
Total requirements during the period to be covered: R
Number of units, ordered every time : q
Costs proportional to q may be neglected, because we must buy R units. The remaining relevant costs are

setup costs for ordening: $\frac{R}{q} \cdot C_o$

stockholding costs : $C_1 \cdot \frac{1}{2} q \cdot \frac{R}{r}$ (cf fig. 2.1)

total costs : $C_o \cdot \frac{R}{q} + \frac{1}{2} C_1 \frac{R}{r} q$ (cf fig. 2.2)

*) The derivations in this section are not strict from the mathematical point of view.

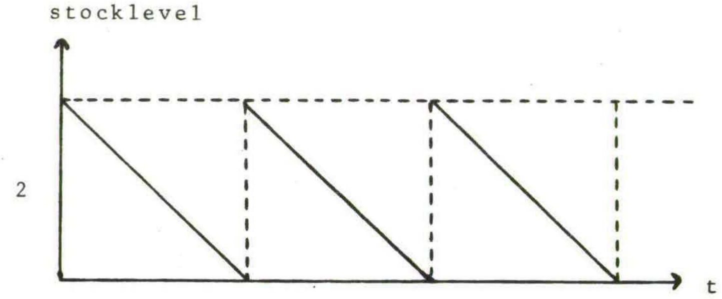

Fig. 2.1

Stocklevel in the course of time in model 1.

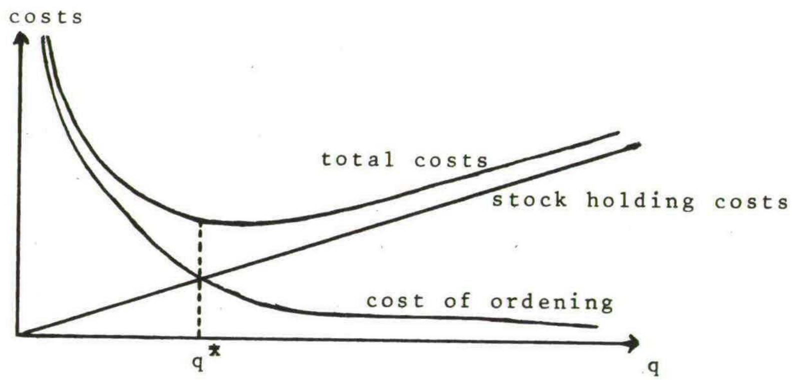

Fig. 2.2

Costs in model 1 as a function of the quantity ordered every time.

We can find the minimum of the total costs either by inspection of graph 2.2 or bij differentiating these costs as a function of q.
In the last case we find, derivate of total costs:

$$-\frac{C_0 R}{q^2} + \frac{1}{2} C_1 \frac{R}{r}.$$

If we put this derivative equal to zero, we find for the optimal value q^* of q

$$q^* = \sqrt{\frac{2 C_0 r}{C_1}} \tag{2.1}$$

This is the classical economic lot size formula, first derived by Harris (1915). The formula is very often used, though the underlying assumptions are rather unrealistic from a practical point of view.

Model 2

In this cáse we make the following assumptions:

1. free
2. irrelevant; e.g. the case in which all demands should be fulfilled and every order should imply a fixed number of units (=q)
3. constant, equal to 1 time units
4. C_1 per unit for every unit which is still in stock if new goods arrive, so costs which could be avoided
5. variable during leadtime: $\underline{r}$; density function of $\underline{r}$: f (r); one demands one unit a time and the demand during leadtime is not larger than q.
6. C_2 per unit short before new goods arrive, so costs which could be avoided; all demands must be fulfilled, if necessary subsequent deliveries.

As to the second question we have no possibilities to choose or to optimize, because the amount to be ordered is given a priori. Let the answer to the first question be: we order as soon as gross stock*) is equal to x. Then the optimal value of x is determined by balancing stock-holding costs against costs of shortages. Two possible situations may arise: a) the number of units in stock is zero and there is demand before the new order arrives; b) the new order arrives and there are still units in stock.

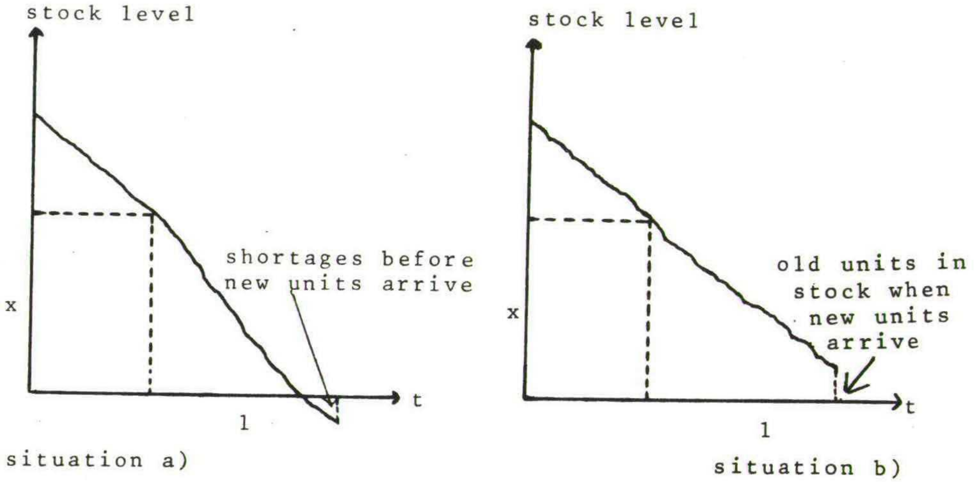

Fig. 2.3

The two possible situations when new units arrive in the case of model 2.

If $\underline{r}$ is equal to a given value r, the costs are respectively:

$$r > x : c_2 \ (r - x)$$

$$r < x : c_1 \ (x - r)$$

---

*) This is not stock on hand, but: stock on hand + stock on orderitems asked for, but not yet delivered.

However, r is stochastic with a density function f(r), so we have to weigh these costs with f(r) and to integrate over all values of r. The total expected costs are:

$$\int_x^{\infty} c_2 \, (r - x) \, f(r) \, dr + \int_o^x c_1 \, (x - r) \, f(r) \, dr$$

The minimum of these total expected costs is found by differentiating to x; the minimizing value x^* of x satisfies:

$$- \int_{x^*}^{\infty} c_2 \, f(r) \, dr + \int_o^{x^*} c_1 \, f(r) \, dr = 0$$

or

$$- c_2 \, [1 - F(x^*)] + c_1 \, F(x^*) = 0.$$

So

$$F(x^*) = \frac{c_2}{c_1 + c_2}, \quad \text{or} \quad 1 - F(x^*) = \frac{c_1}{c_1 + c_2} \qquad (2.2)$$

Thus the reorderpoint or the reorder level x should be chosen in such a way that the probability of demand > x during leadtime is equal to $\frac{c_1}{c_1 + c_2}$, or abbreviated: equal to ε $(= \frac{c_1}{c_1 + c_2})$.

The above derivation also holds if leadtime is variable: then f(r) should again represent the density function of demand during leadtime. If we only know the distribution function of demand per unit of time, then f(r) can (theoretically) be found by integration over the density function of the leadtime.

3. The application of the derived formulae.

From the mathematical point of view it is not difficult to construct models for inventory problems which start from much more realistic assumptions than the simple exemples given in section 2. The models will become more complicated and as a consequence the same holds true for the formulae from which q^* and x^* can be found. Notwithstanding the fact that there are no mathematical diffulties, we usually don't use these more complicated models because of the following reasons:

1. it takes more effort to find q^* and x^* and certainly in the case of many different articles this readily becomes too expensive;
2. it can be shown that the simple formulae given above for q^* and x^* are in most cases not too bad approximations for the formulae one finds from more complicated models;
3. in practice it usually is not possible to estimate all relevant constants very precisely and if this is true it doesn't have much sense to apply complicated formulae which only improve the results theoretically.

   Therefore, in most practical situations the two formulae given in section 2 imply a reasonable compromise between the resemblance of model with reality and the practicability of the results. From the mathematical point of view the combination of the two formulae is very unsatisfactory, because the formulae for q^* is only valid if there is <u>no leadtime</u>, whereas the formula for x^* presupposes <u>a positive leadtime</u>.

For the remainder of our discussion we assume that we use the formulae (2.1) and (2.2) to determine q^* and x^*. We then wait until gross stock has a value $\leq x^*$ and as soon as this happens we order an amount q^*.

4. Computerized administrative systems.

In order to apply an ordering system as described in section 3, we necessarily need an administrative system, telling us continually, how many units are contained in gross-stock. This administrative system may be a list in the storehouse itself, kept up to date by the people withdrawing items from stock, or a listing in an administrative department. There are many examples of both systems, operating reasonably well. But, since the early sixties, many firms (both factories and wholesale trades) in The Netherlands are passing to administrations on smaller or bigger computer systems. Besides irrational arguments the two main reasons are:

1. administrative work done manually is becoming more and more expensive, and
2. if one proceeds to a computerized system a lot of things can be done in just one run of the computer.

Suppose e.g. that we pass all mutations concerning gross-stock to the computer, so all new orders being placed at suppliers, every arrival of goods at the storehouse, every withdrawal from our stocks, may be every transfer from one stock in our firm to another one, may even be every correction, which has to be made. Moreover we include other relevant data, like purchasing price, selling price, serial number of the article, code of supplier, code of customer, and so on. Then a lot of things can be done by the computer:
it can tell us the number of units in stock,
it can tell us the number of units ordered, but not yet delivered,
it can tell us the number of units sold, but not yet shipped,
compare a promised leadtime with real leadtime,
compare a price agreed upon with a price charged,
it can produce lists of invoices to be sent to customers,
it can tell us which invoices of suppliers can be made payable,
and, if necessary, many things more.

It is clear that if one has already such an administrative system on a computer, there are great opportunities to install a systematic inventory system and that this can be done with a relatively small amount of additional effort. The number of units in gross-stock can be compared every week or every day with the reorderpoint x^* and so we can produce a list of articles to be ordered in a particular week or on a particular day. If the optimal quantities to be ordered are also stored in the memory of the computer, we can also present them on the list of goods to be ordered, which makes us independent of clerical work in storehouses or administrative departments. But a computerized system can offer us much more. The greatest difficulty in operating these automatic inventory management systems is to keep our estimated constants reasonably up to date.

If we use the formulae (2.1) and (2.2) for q^* and x^*, this includes the constants r, c_o, c_1, c_2 and the distribution function of demand during leadtime. In the next section we will discuss how to estimate these constants and how to keep them up to date.

5. Estimating and updating the relevant constants.

The setup costs c_o of a new order.

These costs may include:

- costs to report that an order should be placed
- costs of the purchasing department
- (sometimes) transportations costs
- (partially) the costs of putting units received into the storehouses
- costs of administration
- costs of paying the goods.

In most cases these costs are not difficult to find; a good estimate can be found e.g. by dividing the yearly costs of the relevant departments through the number of orders placed. The variation in c_o is usually not very fast and estimating these costs once a year might keep the estimation sufficiently up to date.

The costs c_1 of having one unit during one time unit in stock.

These costs can usually not be estimated for every item separately. A good method to estimate these costs is to relate them to the purchasing price of the article. One then estimates the average purchasing value of the stocks during a year and the total costs made to have the goods stocked. The total costs expressed as a fraction of the average value stocked, will be denoted by i. If the purchasing price of an item equals p, then c_1 is estimated by $\frac{1}{12}$ p i if the time unit is chosen to be a month, so

$$c_1 = \frac{1}{12} p\ i \qquad (5.1)$$

Total stocking costs may include:

costs of storing facilities: maintenance, and depreciation of buildings, light, heating,

maintenance and depreciation of furniture, like cupboards and racks

interest of the capital invested in the facilities

costs of stored goods : costs of repair

depreciation of the goods

interest of the capital invested in the stored goods

assurances and sometimes

taxes

wages of the people running the storing facilities.

In The Netherlands it is not unusual to find values between 0,15 and 0.25 for i. Updating this value one a year suffices in most cases. The value of p can be updated after every new delivery of the supplier.

The costs c_2 of a shortage.

It may be very difficult to find a reasonable value for c_2. A lowerbound is clearly gross profit lost, if there is no unit available and one needs one. Sometimes estimations can be found from contractual duties. If no reasonable estimation is available, one might realize that only the quotient

$\frac{c_1}{c_1 + c_2}$ is necessary, so only the ratio between c_1 and c_2.

Yet another means is, to realize that:

$$\varepsilon = \frac{c_1}{c_1 + c_2}$$

estimates the probability of more demand during leadtime than the number of units available at the reorder level; in this way $1 - \varepsilon$ can be interpreted as a service level to the own factory or to the customers.
It is not uncommon to learn from inventory-managers that they prefer ε -values of about 0.01; it then often turns out impossibly to keep such a low level because of capital restrictions. Values of ε in the neighbourhood of 0.05 might then turn out to be more realistic.

The three constants c_o, c_1 and ε may be different for different groups of articles or products, they usually remain constant for rather long time-periods. On the contrary r and the distribution function of demand during leadtime may vary rather rapidly and then we may make big mistakes if we are working with values being out of date. So we need fast methods to forecast new values for these data.

<u>The average demand r per time-unit.</u>

A classical method to forecast demand is using a moving average, e.g. a twelve months moving average and extrapolating this moving average. This method has two disadvantages: it reacts only slowly if there are changes and a large number of data should be available during a long time, which might be prohibitive in the case of a large number of different products.

A better method is called <u>exponential smoothing</u>. We shall illustrate this method for a simple case to forecast r. Let time be divided into periods of equal length, say into months; period T_t runs from $t - 1$ to t (of Fig. 5.1).

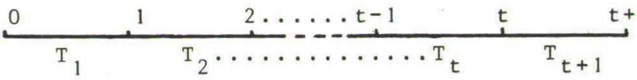

Fig. 5.1

Time-axis divided into periods of equal length

Suppose estimated demand for period T_t is $\hat{r}_t$; at the end of period T_t we know real demand, say r_t. We now estimate demand for period T_{t+1} with a weighted average of $\hat{r}_t$ and r_t:

$$\hat{r}_{t+1} = a\ r_t + (1-a)\ \hat{r}_t \tag{5.2}$$

in which a is a constant, satisfying $0 \leqq a \leqq 1$. If we express $\hat{r}_t$ in the same way as an average of $\hat{r}_{t-1}$ and r_{t-1}, we find

$$\hat{r}_{t+1} = a\ r_t + (1-a)\ a\ r_{t-1} + (1-a)^2\ \hat{r}_{t-1}\ ;$$

and continuing in the same way, we find:

$$\hat{r}_{t+1} = a \sum_{k=0}^{t-1} (1-a)^k\ r_{t-k} + (1-a)^t\ \hat{r}_1 . \tag{5.3}$$

This expression means that <u>lower weights</u> are given to older observations, but that is just what we would like to do intuitively. Besides this advantage computing $\hat{r}_{t+1}$ from r_t and $\hat{r}_t$ is very fast; if we rewrite the formula as

$$\hat{r}_{t+1} = \hat{r}_t + a\,(r_t - \hat{r}_t), \tag{5.4}$$

we have only one multiplication and two additions. Furthermore we need only the value of a and the last values of $\hat{r}_t$ and r_t to find $\hat{r}_{t+1}$. Summarizing, the advantages of exponential smoothing are:

1) lower weights to more remote observations
2) very fast, especially in the form (5.4)
3) the method uses only limited storage capacity
4) if one has made mistakes, the influence of them is gradually dying out.

About the value of a, to be chosen, we might recommend a rather small value in not too irregular situations e.g. 0.1 or 0.2. The difference in weights given to past observations can easily be found. If data come in, let us say quarterly, then a value of a equal to 0.5 might be better, because otherwise we are giving too high weights to rather remote historical data.

The computer can make these forecasts automatically because we assumed all mutations in gross-stock to be communicated to it, so r_t is known for every period.

The exponential smoothing formula (5.2) is the simplest one; for situations in which a trend or seasonal fluctuations should be taken into account, there exist other, more compicated methods to do the forecasts (cf. R.G.Brown (1)).

Demand during leadtime.

In order to compute the reorder point with formula (2.2) we need the distribution function of demand during leadtime. Now, in most cases leadtime is variable and moreover demand during

a constant timeperiod is variable. Theoretically we can try to find both distributionfunctions and derive the distribution function F of demand during leadtime from them. In practice this is hardly ever possible, even if we have only a limited number of different articles.
A rather primitive method may help sometimes. We mark ordertimes and arrivaltimes of the corresponding goods on the cards in the storehouse or in the administrative system, to be able to estimate demand during leadtime and then next try to find x^*.
A drawback of this system is the complicated administrative procedure and the difficulty to fit good distribution functions F to the data. It can be done on a computer, but then the last mentioned difficulty remains, whereas it also takes much of the capacity of the computers memory.
More succesful methods are based on the consideration that in cases of very low demand during leadtime a Poisson-distribution usually gives a good fit to the observations, whereas in all other cases a gamma distribution can be used. Here we restrict ourselves to the case in which a gamma distribution is fitted to the observations.
The general expression for the density function of a γ-distribution is

$$f(x) = \frac{\lambda^{\alpha}}{\Gamma(\alpha)} e^{-\lambda x} x^{\alpha-1} \qquad \lambda \text{ and } \alpha > 0 \tag{5.5}$$

For different values of α we get completely different forms of f(x), cf. Fig.5.2.

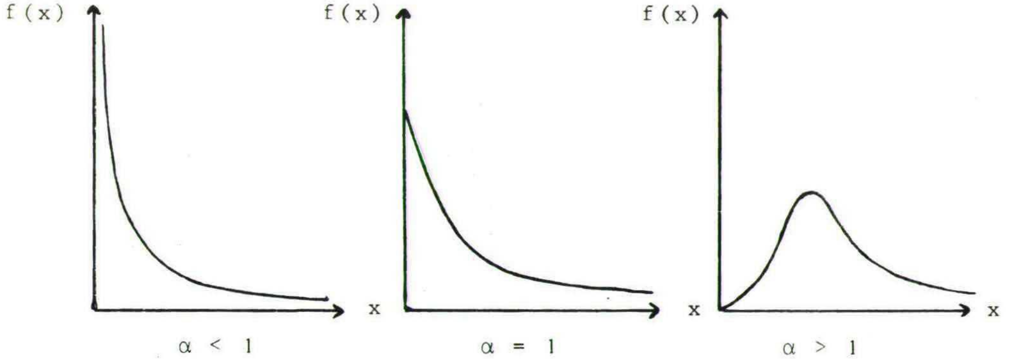

Fig. 5.2

Different shapes of the density function f(x) for different values of α.

For α an integer k, we get in stead of (5.5)

$$\frac{\lambda^k}{(k-1)!} e^{-\lambda x} x^{k-1}, \tag{5.6}$$

the so-called Erlang distribution.
Let us restrict ourselves to the case in which F can reasonably be approximated by an Erlang distribution. Then (2.2) changes into

$$\int_{x^\star}^{\infty} \frac{\lambda^k}{(k-1)!} e^{-\lambda x} x^{k-1} dx = \frac{c_1}{c_1+c_2} = \varepsilon \tag{5.7}$$

From this formula $x^\star$ can be found if k and ε are known. The computations can be easily done if we make the following transformations.

Substitute for λx the variable y, then (5.7) transforms into

$$\int_{\lambda x^*}^{\infty} \frac{1}{(k-1)!} e^{-y} y^{k-1} \, dy = \varepsilon$$

so, λx^* is a function of ε and k, to be denoted by g_1:

$$\lambda x^* = g_1 (\varepsilon, k).$$

Thus

$$x^* = \frac{1}{\lambda} g_1 (\varepsilon, k). \tag{5.8}$$

If demand during leadtime is $\underline{r}_L$, then

$$\mathcal{E}\, \underline{r}_L = \frac{k}{\lambda} \quad \text{and} \quad \sigma^2 (\underline{r}_L) = \frac{k}{\lambda^2}.$$

The coefficient of variation of $\underline{r}_L$ equals

$$V (\underline{r}_L) = \frac{\sigma(\underline{r}_L)}{\mathcal{E}\, \underline{r}_L} = \frac{\sqrt{k}/\lambda}{k/\lambda} = \frac{1}{\sqrt{k}} .$$

Substituting $\mathcal{E}\, \underline{r}_L$ and $V (\underline{r}_L)$ into (5.8) leads to

$$x^* = \mathcal{E}\, \underline{r}_L \cdot \frac{1}{k} \cdot g_1 (\varepsilon, k) = \mathcal{E}\, \underline{r}_L \cdot V^2(\underline{r}_L) \cdot g_1 \left(\varepsilon, \frac{1}{V^2(\underline{r}_L)}\right) \tag{5.9}$$

or

$$x^* = \mathcal{E}\, \underline{r}_L \cdot g_2(\varepsilon, V(\underline{r}_L)).$$

If we know ε and $V(\underline{r}_L)$, the value $g_2(\varepsilon, V(\underline{r}_L))$ can be found if g_2 has been tabulated, which has been done by VAN HEES (2), cf the appendix. <u>Then x^* equals a given multiple of the expected demand during leadtime.</u>
The evaluation of ε has been discussed earlier, so that we now can restrict ourselves to the evaluation of

$$V(\underline{r}_L) = \frac{\sigma(\underline{r}_L)}{\mathcal{E}\, \underline{r}_L}.$$

If we denote demand for one period by $\underline{r}$, the expected value by $\mathcal{E}(\underline{r})$ and its variance by $\sigma^2(\underline{r})$, if we assume leadtime being L time periods and the demands during these periods mutually independent, then we find

$$\mathcal{E}\, \underline{r}_L = L.\ \mathcal{E}\, \underline{r} \tag{5.10}$$

and

$$\sigma(\underline{r}_L) = \sqrt{L}\ .\ \sigma(\underline{r})\ ; \tag{5.11}$$

thus

$$V(\underline{r}_L) = \frac{\sqrt{L}\ .\ \sigma(\underline{r})}{L\ .\ \mathcal{E}\ \underline{r}} = \frac{1}{\sqrt{L}}\ .\ \frac{\sigma(\underline{r})}{\mathcal{E}\ \underline{r}} = \frac{1}{\sqrt{L}}\ .\ V(\underline{r}). \tag{5.12}$$

We illustrate by a numerical example. Assume:

$$\mathcal{E}\ \underline{r} = 280 \quad , \quad \sigma(\underline{r}) = 200$$
$$L = 2 \quad , \quad \varepsilon = 0{,}05\ .$$

Then

$$V(\underline{r}_L) = \frac{1}{\sqrt{2}} . \frac{200}{280} = \frac{1}{1.4}\ .\ \frac{10}{14} \approx 0.5$$

$$g_2\ (0.05\ ;\ 0.5) = 1.94 \quad \text{(cf.appendix)}$$

and

$$x^{*} = \& \underline{r}_L \cdot g_2 \ (0.05 \ ; \ 0.5) = 2.\ 280.(1.\ 94) \simeq 1120.$$

In the case of a stochastic leadtime (5.10) should be replaced by $\& \underline{r}_L = \& \underline{L} \ . \ \& \underline{r}$, whereas (5.11) gives an underestimation of $\sigma \ (\underline{r}_L)$ and also (5.12) is an underestimation of $V \ (\underline{r}_L)$, which can be improved theoretically, but in practice usually not.

From (5.12) it follows that x^{*} can be computed if we know $V \ (\underline{r})$ and L. How to estimate $\& \underline{r}$, formely denoted by r, has been discussed before. Up to date estimations of L can be found by computations, similar to formula (5.4). Every time we have a new date about leadtime, we make a new estimation of the expected leadtime, using the formula

$$\hat{L}_n = a \ L_0 + (1-a) \ \hat{L}_0 + a \ (L_0 - \hat{L}_0) \ ; \qquad (5.13)$$

$\hat{L}_n$ being newly expected leadtime

L_0 being last observed leadtime

$\hat{L}_0$ being last forecasted leadtime.

If new orders are only placed every two or three months, then values of a about 0.5 might be very usuful; even a value of a = 0.2 would mean that leadtimes of more then 2 years ago are given a weight of about 10% which doesn't seem very reasonable.

It remains to estimate $\sigma \ (\underline{r})$. In principle several different methods are available.

The second moment $\& \underline{r}^2$ can be updated in the same way as the first one and from the updated first and second moments we can derive $\sigma^2 \ (\underline{r})$ and $\sigma \ (\underline{r})$. Disadvantages of this method are: much computational effort, great use of storing facilities of the computer and a great relative inaccuracy in the estimation of $\sigma^2 \ (\underline{r})$.

A second method is to update the first absolute moment Δ and then to use the relation

$$\frac{\Delta\ (\underline{r})}{\sigma\ (\underline{r})} = \int_{-\infty}^{+\infty} |u|\ g(u)\ du, \qquad (5.14)$$

in which g(u) is the density function of demand per timeperiod. The integral can be evaluated if we know g(u); if the distribution function of demand per timeperiod is also an Erlang-distribution, then the quotient $\frac{\Delta\ (\underline{r})}{\sigma\ (\underline{r})}$ varies between 0.74 and 0.80, whereas for all values of $k \geqq 5$, 0.79 is a reasonable estimate of $\frac{\Delta\ (\underline{r})}{\sigma\ (\underline{r})}$ and $\sigma\ (\underline{r})$ can be found from

$$\sigma\ (\underline{r}) \simeq 1.27\ \Delta\ (\underline{r}) \qquad (5.15)$$

If the shape of g(u) is only badly known, this method is very limited in its practical application.
A third method corresponds to the linear relationship between log σ and log $\mathcal{E}$ for many distribution functions, e.g. the Poisson and the γ-distribution. These relations are easily derived as follows:

Poisson: $\mathcal{E}\ \underline{r} = \lambda\ ,\ \sigma(\underline{r}) = \sqrt{\lambda} \rightarrow \log \sigma(\underline{r}) = \frac{1}{2} \log \mathcal{E}\ \underline{r}$

gamma : $\mathcal{E}\ \underline{r} = \frac{\alpha}{\lambda}\ ,\ \sigma(\underline{r}) = \frac{\sqrt{\alpha}}{\lambda} \rightarrow \log \sigma(\underline{r}) = -\frac{1}{2} \log \alpha + \log \mathcal{E}\ \underline{r}$.

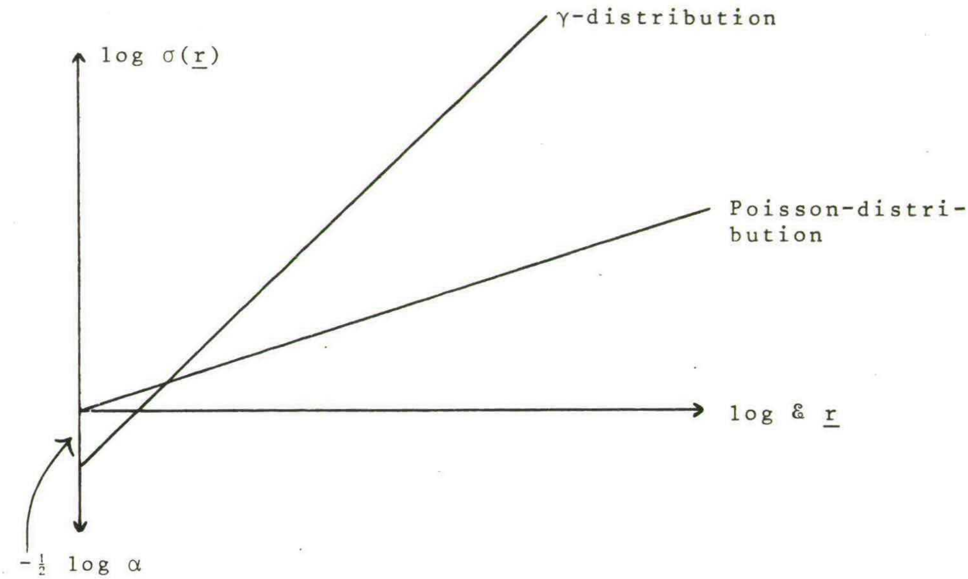

Fig. 5.3

Graph of the relation between log $\sigma(\underline{r})$ and log & $\underline{r}$ for the γ- and the Poisson distribution.

Empirical investigations also show that if we take groups of articles of the same type, we often find a very good linear relationship between log σ and log &. If this holds true, we can estimate the constants of the relationship by the method of least squares and then find a forecast of log σ if we have forecasted log & $\underline{r}$.
Next $V(\underline{r})$ is easily found and if we have a forecast for L, the same holds true for $V(\underline{r}_L)$ (formula (5.10)) and subsequently also for & $\underline{r}_L$ and x^*.

In the last system we only have to update regularly the price p, & $\underline{r}$ with formula (5.4) and L with formula (5.13), so p, & $\underline{r}$ and L; the other data might be reviewed e.g. once a year. Doing this, it is possible to regularly produce up to data values of x^* and of goods to be ordered with the corresponding

ordering quantities q^*.
Nevertheless it remains desirable to continue visual checks of the automatically suggested ordering proposals, because of e.g. incidental selling activities, products to be taken out of circulation, articles for which the system leads to severe mistakes, and so on. A number of completely worked out examples of the installment of systematic inventory management systems are given in the book by BUCHAN and KOENIGSBERG (3).

References.

(1) R.G.BROWN, Statistical Forecasting for Inventory Control, Mc Graw-Hill Book Company, New York (1959).

(2) R.N. van HEES, Bestelniveau- en serie-grootte, Tijdschrift voor Efficiëntie en Documentatie 31 (1961) 63 - 69.

(3) J.BUCHAN and E.KOENIGSBERG, Scientific Inventory Management, Prentice Hall Inc. Englewood Cliffs (1963).

A.K.

APPENDIX

Table of values of the function g_2 (ε, $V(\underline{r}_L)$)*)

$V(\underline{v}_L)$	ε					
	0,10	0,05	0,025	0,01	0,005	0,001
1,40	2,69	3,80	4,99	6,56	7,77	10,60
1,35	2,64	3,70	4,81	6,31	7,47	9,82
1,30	2,59	3,60	4,66	6,06	7,17	9,32
1,25	2,54	3,50	4,50	5,81	6,86	8,83
1,20	2,49	3,39	4,34	5,56	6,55	8,40
1,15	2,44	3,29	4,18	5,32	6,24	8,00
1,10	2,39	3,19	4,02	5,08	5,93	7,62
1,05	2,34	3,08	3,86	4,84	5,62	7,25
1,00	2,29	2,98	3,71	4,61	5,31	6,88
0,95	2,24	2,88	3,56	4,38	5,00	6,50
0,90	2,18	2,77	3,41	4,15	4,72	6,13
0,85	2,12	2,66	3,25	3,93	4,46	5,75
0,80	2,06	2,56	3,10	3,72	4,21	5,38
0,75	2,00	2,46	2,95	3,51	3,93	5,00
0,70	1,94	2,36	2,80	3,30	3,68	4,62
0,65	1,87	2,25	2,64	3,10	3,44	4,20
0,60	1,81	2,15	2,49	2,90	3,20	3,90
0,55	1,74	2,04	2,34	2,70	2,97	3,58
0,50	1,67	1,94	2,19	2,51	2,75	3,27
0,45	1,60	1,84	2,05	2,33	2,53	2,97
0,40	1,54	1,74	1,92	2,16	2,35	2,68
0,35	1,47	1,64	1,80	1,99	2,16	2,43
0,30	1,40	1,54	1,67	1,83	1,97	2,18
0,25	1,34	1,44	1,55	1,67	1,79	1,95
0,20	1,28	1,35	1,43	1,52	1,62	1,73
0,15	1,21	1,26	1,31	1,38	1,45	1,53
0,10	1,13	1,17	1,20	1,24	1,29	1,34
0,05	1,06	1,08	1,10	1,12	1,13	1,16

*) This table is reprinted from R.N.VAN HEES, Tijdschrift voor Efficiëntie en Documentatie 31 (1961) p. 66.

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